Asymptotically Tight Bounds on the Time Complexity of Broadcast and its Variants in Dynamic Networks

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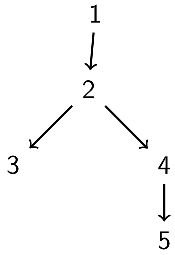
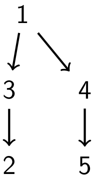
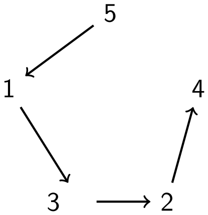
ITCS 2023



Round 1



Round 2

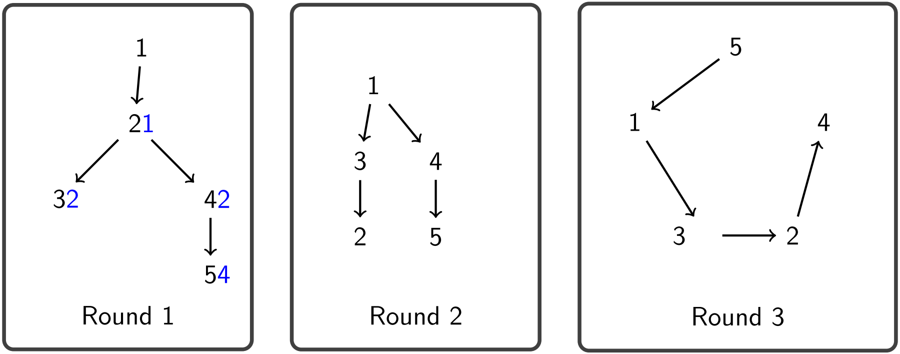


Round 3

Information Dissemination in Dynamic Rooted Trees

- The network of each round can be a different rooted tree.
- Each node transmits all I.D.s it has received in previous rounds.

Information Dissemination In Dynamic Networks



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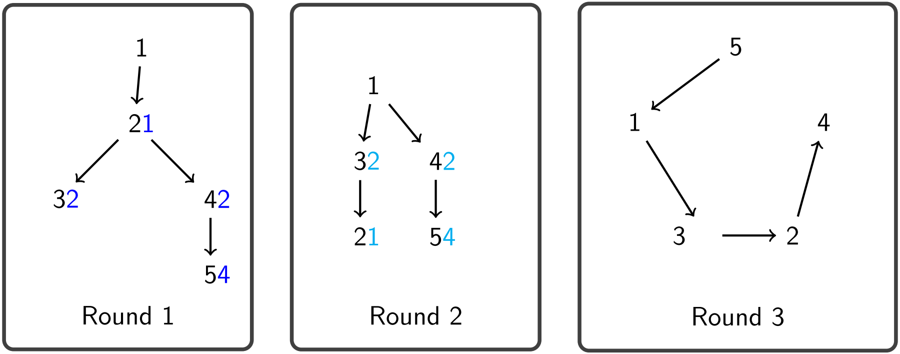

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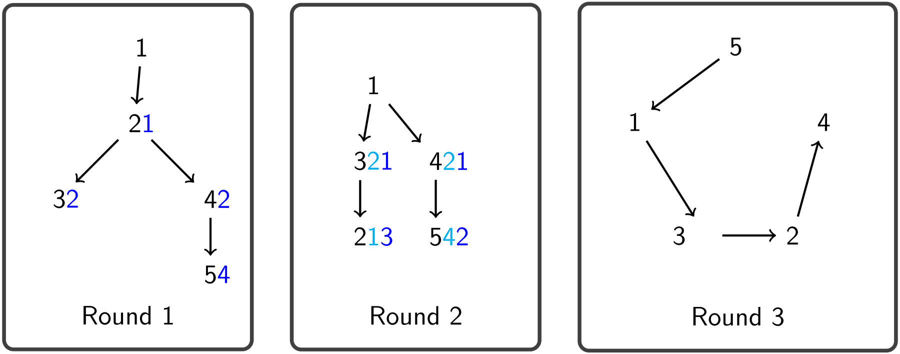

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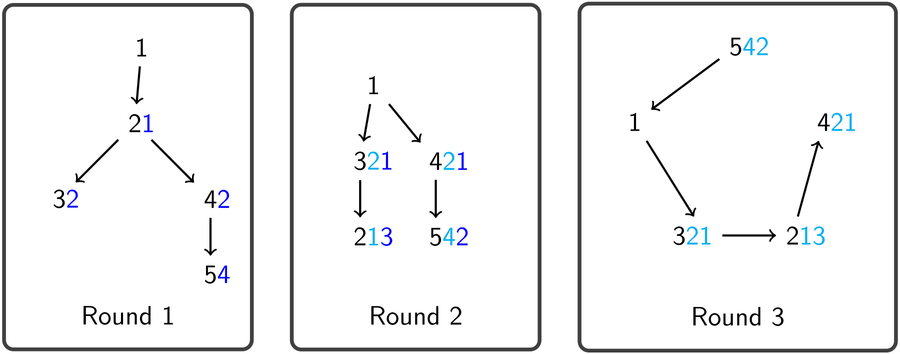

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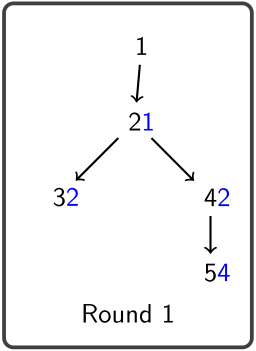

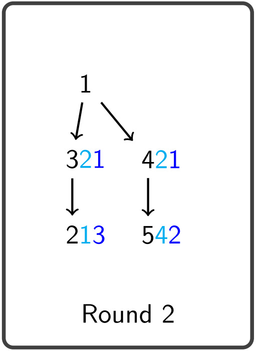

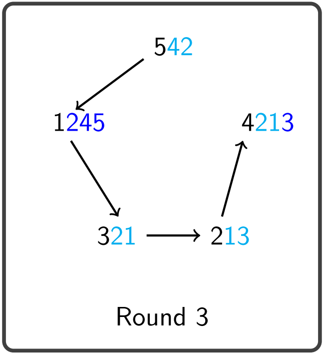

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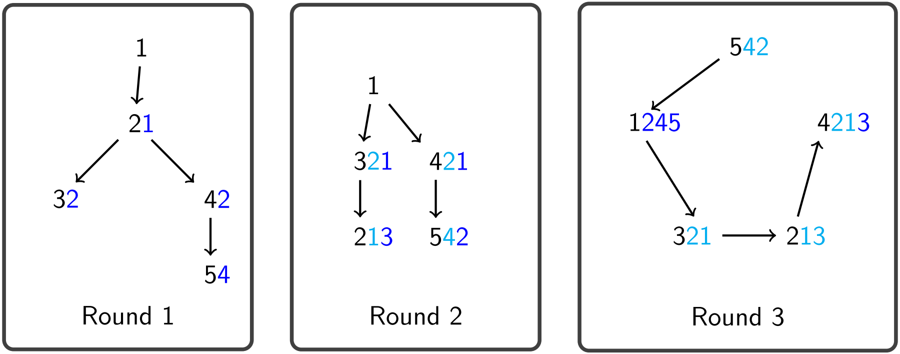

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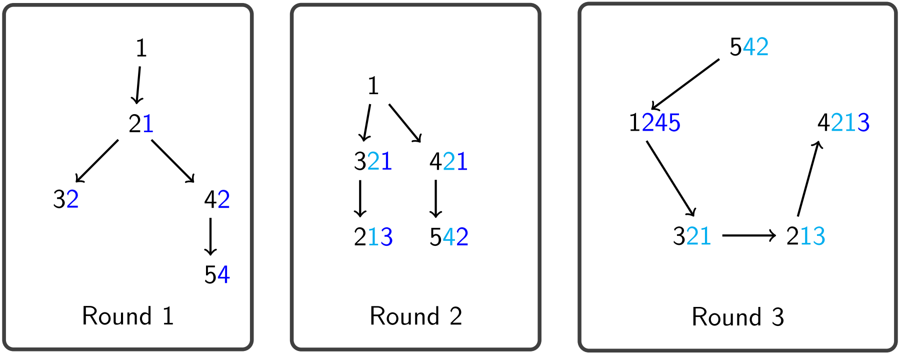
- Broadcast is when 1 I.D. reaches everyone

Information Dissemination In Dynamic Networks



Information Dissemination in Dynamic Rooted Trees

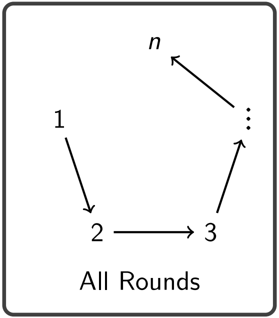
- The network of each round can be a different rooted tree.
- Each node transmits all I.D.s it has received in previous rounds.
- Broadcast is when 1 I.D. reaches everyone
- How many rounds do we need to ensure Broadcast?

Adversarial Model

Adversarial Model

- An adversary can choose any network among a set A of predefined networks.
- There's an objective the adversary tries to delay as much as possible.
- We want to determine the number of rounds T the adversary can delay the objective.

Example for $n-1$ rounds:



Previous Work

- [Charron-Bost, Schiper ‘09] + [Charron-Bost, Függer, Nowak ‘15] : $O(n \log n)$.
- [Zeiner, Schwarz, Schmid ‘19] : $O(n \log n)$ (General Case); $O(kn)$ if k internal nodes or k leaves in each round.
- [Függer, Nowak, Winkler ’20] :$O(n \log \log n)$.

Our Work: $\theta(n)$

Main intuitions

Main Observation

Any I.D. received by the root before the start of a round, is received by at least one new process during the round.

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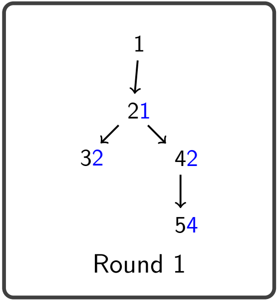

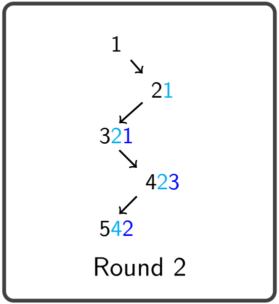

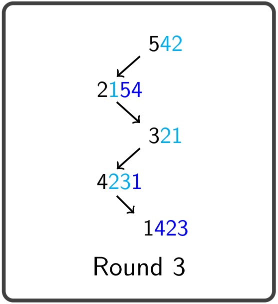

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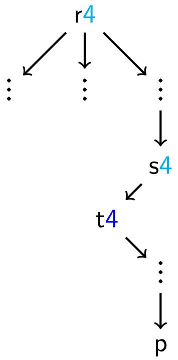


- If an I.D. has been received by n roots, then everyone has received the I.D.
- We will keep track of the I.D.s the root has received before each round.

Broadcast

Create a new graph:

- one node for each I.D.
- one node for each round.

For each round t, add an edge from every I.D. the root has received, and from every round $t' < t$ if the root of t has received the I.D. of the root of t'.

I.D.s

1

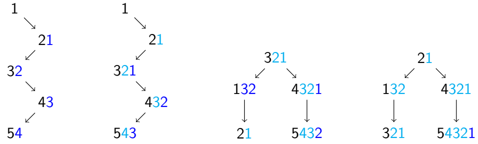
2

3

4

n=5

rounds	1	2	3	4	⋯	$3n$
root	1	1	3	2	⋯	



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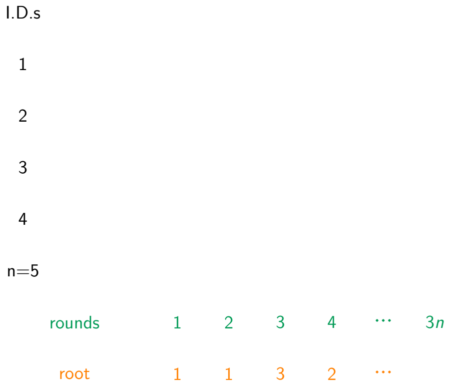

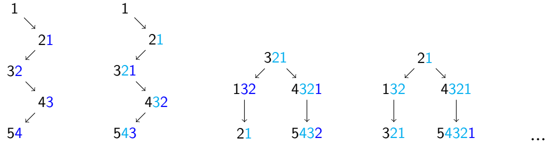

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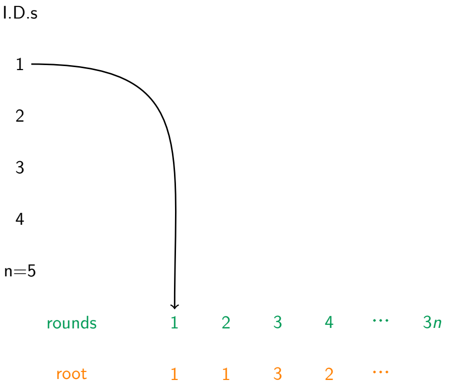

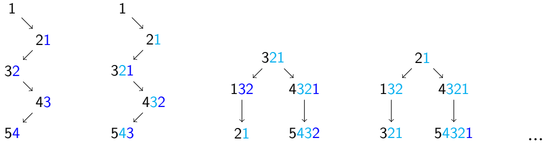

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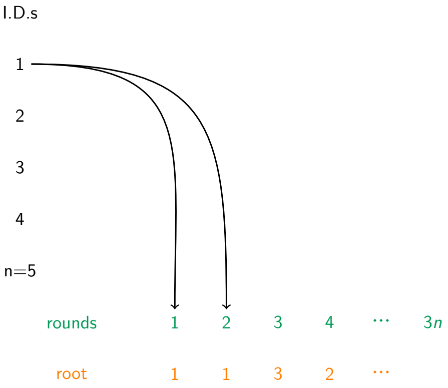

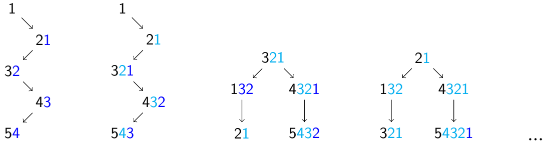

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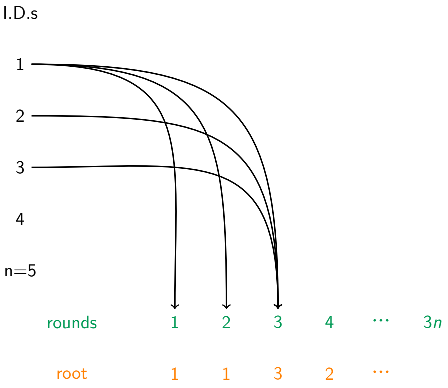

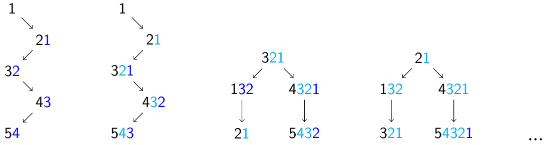

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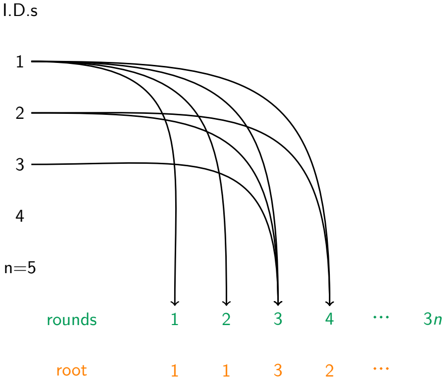

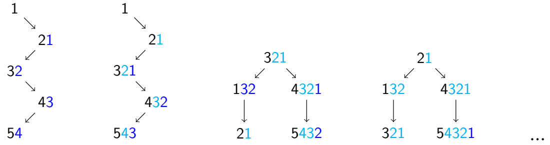

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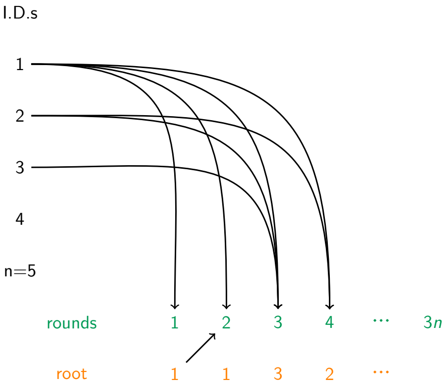

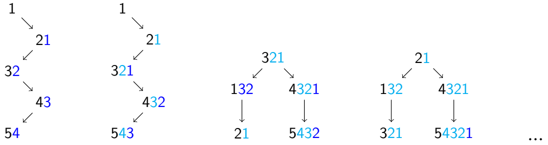

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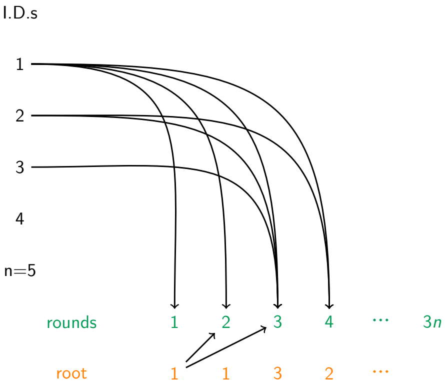

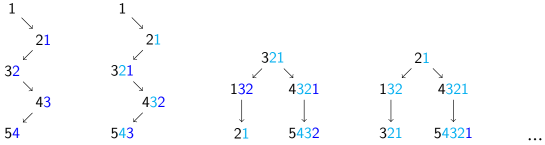

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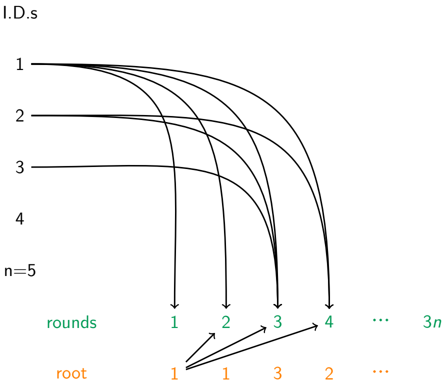

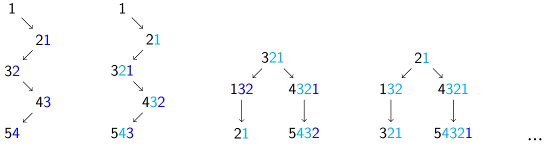

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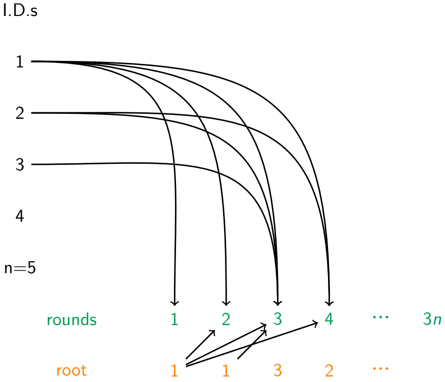

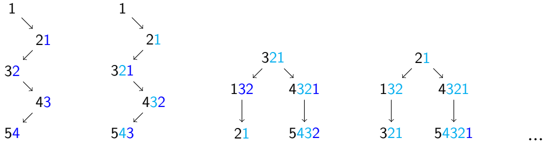

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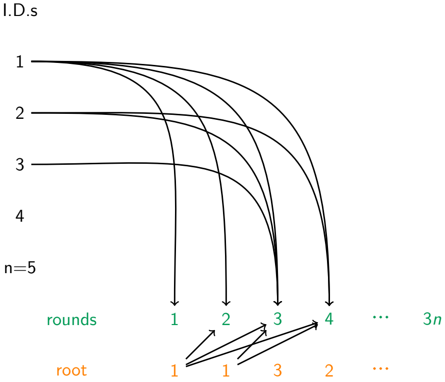

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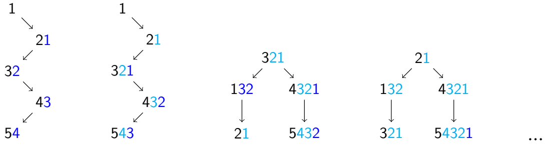

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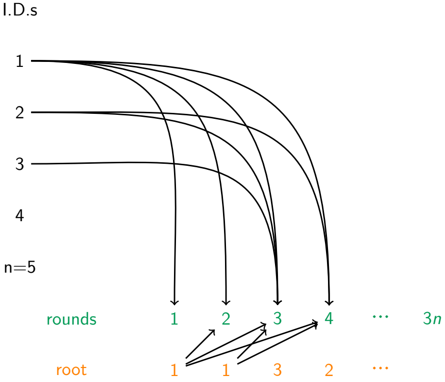

Observations:

- If a node has degree at least n, then the corresponding $I.D.$ has reached everyone.
- Round t has in-degree at least t.
- The total number of edges is larger than $\sum_{t=1}^{3n} t = \frac{9n^2}{2}$.
- We have $4n$ nodes total

Broadcast

The Upper Bound

An upper bound for Broadcast on rooted trees is $O(n)$.

Broadcast

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An upper bound for Broadcast on rooted trees is $O(n)$.

The Lower Bound

A lower bound for Broadcast on rooted trees is $\Omega(n)$ [a].

[a] Zeiner, M., Schwarz, M., and Schmid, U. (2019). On linear-time data dissemination in dynamic rooted trees. Discrete Applied Mathematics, 255, 307-319.

k-Broadcast

k-Broadcast on k-Rooted Networks

- A: the set of networks on n processes with k roots.
- Objective: k I.D.s that has each been received by everyone.
- We prove $T = \Theta(n)$.

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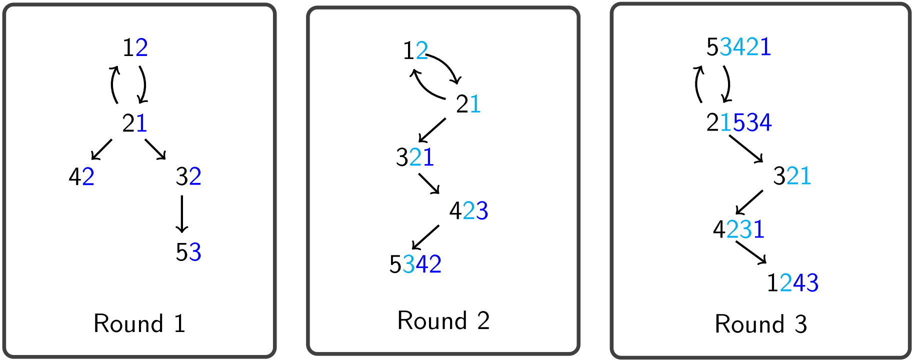

2-Broadcast in 3 rounds. Broadcasters: 1 and 2.

k-Broadcast

The Upper Bound

An upper bound for k-Broadcast on networks with k roots is $O(n)$.

k-Broadcast

The Upper Bound

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The Lower Bound

A lower bound for k-Broadcast on networks with k roots is $\Omega(n)$.

k-Cover

Cover of size k on k-Forests

- A: the set of forests on n processes with k rooted trees.
- Objective: k I.D.s such that everyone has received at least one of them.
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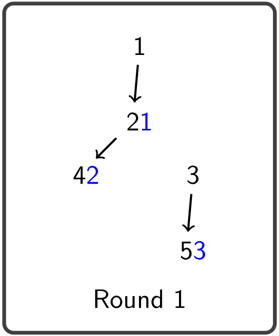

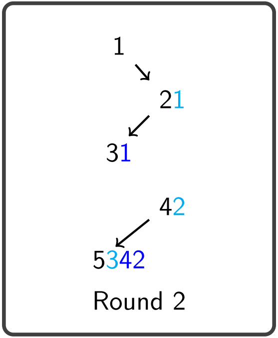

2-Cover in 2 rounds.
Coverers: 1 and 2.

Results

The Upper Bound

An upper bound for Cover of size k on k-forests is $O(n)$.

Results

The Upper Bound

An upper bound for Cover of size k on k-forests is $O(n)$.

The Lower Bound

A lower bound for Cover of size k on k-forests is $\Omega(n - k)$.

Takeaway and Future Directions

Main Takeaway

In the worst case scenario, when enough connectivity is ensured and when there is no limit on the message sizes, data dissemination is linear.

Future Work:

- Find ways to speed up the objectives by constraining the adversary differently.
- Look at a random adversary rather than a “smart” one.
- Look at applications - Leader election or Consensus.
- Look at message size constraints.