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Reconfiguration of homomorphisms.

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The first part of the talk is joint work with Benjamin Lévêque (CNRS). The second part is joint work with Mark Siggers (Kyungpook National University, South Korea).

Given two graphs G et H, the recoloring graph $\mathrm{Hom}_1(G, H)$ is a graph whose vertices are homomorphisms $G \to H$ (also called "H-colorings") and two homomorphisms are neighbors if and only if they differ on exactly one vertex. The graph $\mathrm{Hom}_1(G, H)$ is has been studied for example in the context of uniform sampling of colorings and local search algorithms. For a fixed graph H, the following problems have been studied :

1. Given a graph G and two homomorphisms $\alpha, \beta : G \to H$, we ask whether there is a path from α to β in $\mathrm{Hom}_1(G, H)$. This is the H-recoloring problem.
2. Given a graph G, is $\mathrm{Hom}_1(G, H)$ connected ? This is the H-mixing problem.

Generalizing previous results, we show the following :

Théorème 1 *H-recoloring is polynomial when :*
- *H is an irreflexive digraph not containing a square of algebraic girth* 0.
- *H is a reflexive digraph not containing a square of algebraic girth* 0 *neither a transitive triangle.*

Théorème 2 *H-mixing is co-NP complete when H is a digraph non tree, not containing a square of algebraic girth* 0 *and such that $H \times A_2$ is connected.*

All these results rely on a topological method introduced in [1]. We give an insight of the key ideas behind this method. Since Theorem 2 also depends on a topological machinery developped in [2], we will show how this machinery works and how we extended it use to irreflexive graphs.

Références

[1] Marcin Wrochna. *Homomorphism reconfiguration via homotopy* SIAM J. Discret. Math., **34(1)** :328–350, 2020.

[2] Hyobeen Kim, Jae baek Lee, and Mark Siggers. Mixing is hard for triangle- free reflexive graphs, 2023. arXiv :2207.03632.