1-extendable partition of graphs

Bergé Pierre, Université Clermont Auvergne, LIMOS, France, `pierre.berge@uca.fr`

Feghali Carl, CNRS, ENS de Lyon, LIP, France, `carl.feghali@ens-lyon.fr`

Marin Malory, ENS de Lyon, LIP, France, `malory.marin@ens-lyon.fr`

Watrigant Rémi, Université Claude Bernard Lyon 1, LIP, France, `remi.watrigant@univ-lyon1.fr`

A graph is said *1-extendable* if each of its vertices belongs to a Maximum Independent Set (MIS). This concept was introduced by Berge and is closely related to the study of well-covered graphs, originally introduced by Plummer. In the context of Wi-Fi networks, 1-extendability plays a crucial role in ensuring equitable network performance [1]. Specifically, in the conflict graph of a Wi-Fi network, the throughput of each access point is proportional to the number of Maximum Independent Sets to which its vertices belong. In simpler terms, achieving 1-extendability in the graph ensures minimal fairness.

In practice, it is possible to assign channels to each access point, resulting in a vertex partition of the conflict graph, such that each channel induces a 1-extendable graph.

In this work, we focus on the problem of partitioning a graph into the fewest possible number of 1-extendable subgraphs. We introduce a new parameter, denoted as $\chi_{\text{1-ext}}(G)$, which represents the minimum order of such a partition of the graph G. We begin by establishing some extremal properties of this parameter, particularly by demonstrating that $\chi_{\text{1-ext}}(G) = \mathcal{O}(\sqrt{n})$, where n is the number of vertices in G. Next, we shift our focus to cographs, revealing that $\chi_{\text{1-ext}}(G)$ follows a more favorable upper bound of $\mathcal{O}(\log_2(n))$. Furthermore, we provide a quasi-polynomial algorithm designed to efficiently compute $\chi_{\text{1-ext}}(G)$ specifically for cographs.

Références

[1] Pierre Bergé, Anthony Busson, Carl Feghali, and Rémi Watrigant, *1-extendability of independent sets.* In Cristina Bazgan and Henning Fernau, editors, *Combinatorial Algorithms*, pages 172–185, Cham, 2022. Springer International Publishing.