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Some problems and results on acyclic sets and colouring of digraphs

Ararat Harutyunyan, LAMSADE, Université Paris Dauphine
Colin McDiarmid, Mathematical Institute, University of Oxford
Gil Puig i Surroca, LAMSADE, Université Paris Dauphine

An *acyclic set* of a digraph D is a set of vertices $S \subseteq V(D)$ such that the induced subdigraph $D[S]$ has no directed cycle. The size of the largest acyclic set of D is denoted by $\vec{\alpha}(D)$. The *dichromatic number* of D, denoted by $\vec{\chi}(D)$, is the minimum number of acyclic sets into which $V(D)$ can be partitioned. The *list dichromatic number* of D, denoted by $\vec{\chi}_\ell(D)$, is the minimum k such that, for any assignment of lists of size k to the vertices of D, it is possible to colour the vertices using colours from their lists in a way that the resulting colour classes are acyclic. We first study these parameters in terms of the *circumference* of D, that is, the length of its largest directed cycle. Theorem 1 is reminiscent of a classical theorem of Bondy [1].

Theorem 1 *Let D be a directed graph with circumference $s \geq 2$. Then $\vec{\chi}_\ell(D) \leq s$.*

For tournaments, the result can be improved. The following bound is sharp up to a constant factor.

Theorem 2 *Let T be a tournament of circumference s. Then $\vec{\chi}_\ell(T) \leq (1 + o(1))s/\log_2 s$ as $s \to \infty$.*

Additionally, we slightly improve a result from [2], which gives a bound for the dichromatic number in terms of the circumference and the digirth. Then, in a second section, we focus on the size of largest acyclic sets. The main result of this section concerns random regular digraphs.

Theorem 3 *Let $r \geq 2$ be an integer and let $D_{n,r}$ be a random digraph, chosen uniformly among all r-regular n-vertex oriented simple graphs with labelled vertices. Then, $\vec{\alpha}(D_{n,r}) = \Theta(n \ln r/r)$ asymptotically almost surely.*

References

[1] Bondy, *Diconnected orientations and a conjecture of Las Vergnas*, J. London Math. Soc. **14(2)**: 277–282 (1976).

[2] Cordero-Michel, Galeana-Sánchez, *New bounds for the dichromatic number of a digraph*, Discret. Math. Theor. Comput. Sci. **21(1)** #7 (2019).