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Recognizing unit multiple interval graphs is hard

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Multiple interval graphs are a well-known generalization of interval graphs introduced in the 1970s to deal with situations arising naturally in scheduling and allocation. A d-interval is the union of d intervals on the real line, and a graph is a d-interval graph if it is the intersection graph of d-intervals. In particular, it is a unit d-interval graph if it admits a d-interval representation where every interval has unit length.

Whereas it has been known for a long time that recognizing 2-interval graphs and other related classes such as 2-track interval graphs is NP-complete [1, 2], the complexity of recognizing unit 2-interval graphs remains open. Here, we settle this question by proving that the recognition of unit 2-interval graphs is also NP-complete. Our proof technique uses a completely different approach from the other hardness results of recognizing related classes. Furthermore, we extend the result for unit d-interval graphs for any $d \geq 2$, which does not follow directly in graph recognition problems –as an example, it took almost 20 years to close the gap between $d = 2$ and $d > 2$ for the recognition of d-track interval graphs. Our result has several implications, including that recognizing $(x, \ldots, x)$ d-interval graphs and depth r unit 2-interval graphs is NP-complete for every $x \geq 11$ and every $r \geq 4$. The main results are summarized below.

Théorème 1 *Recognizing unit d-interval graphs is NP-complete for every $d \geq 2$.*

Corollaire 2 *Recognizing $(x, \ldots, x)$ d-interval graphs and depth r unit 2-interval graphs is NP-complete for every $x \geq 11$ and every $r \geq 4$.*

Références

[1] Douglas B. West and David B. Shmoys, *Recognizing graphs with fixed interval number is NP-complete*, Disrecte Appl. Math., 8 :295–305, 1984.

[2] András Gyárfás and Douglas West, *Multitrack interval graphs*, Congressus Numerantium, pages 109–116, 1995.