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Reconfiguration of plane trees in convex geometric graphs

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For a set C of n points in convex position, a non-crossing spanning tree is a spanning tree of the points where every pair of edges, represented by the straight line interval between their endpoints, are pairwise non-crossing. We investigate the minimum number of flips required to reconfigure a non-crossing spanning tree on C to another using a sequence of flips, where a flip removes an edge then adds a new one such that the result is still a non-crossing spanning tree on C.

The naive upper bound of $2n - 4$ [1] on the minimum number of flips required stood up for 25 years until recent improvements. Indeed, Aicholzer et al. [2] proved a $2\delta - \Omega(\log \delta)$ upper bound, with δ half of the size of the symmetric difference between the two trees. And then, Bousquet et al. [3] proved a $2n - \Omega(\sqrt{n})$ upper bound. We further improve these upper bounds :

Théorème 1 *Let C be a set of n points in convex position. There exists a flip sequence between any pair of non-crossing spanning trees T_1 and T_2 of length at most $c \cdot \delta(T_1, T_2)$ with $c = \frac{1}{12}(22 + \sqrt{2}) \approx 1.95$. In particular, there exists a transformation of length at most $cn \approx 1.95n$.*

Moreover, we also improve the known lower bound of $\frac{3}{2}n - 5$ flips into $\frac{5}{3}\delta$.

Références

[1] David Avis and Komei Fukuda. *Reverse search for enumeration.* volume 65, pages 21–46. 1996. First International Colloquium on Graphs and Optimization (GOI), 1992 (Grimentz).

[2] Oswin Aichholzer, Brad Ballinger, Therese Biedl, Mirela Damian, Erik D Demaine, Matias Korman, Anna Lubiw, Jayson Lynch, Josef Tkadlec, and Yushi Uno. *Reconfiguration of non-crossing spanning trees.* arXiv preprint arXiv :2206.03879, 2022.

[3] Nicolas Bousquet, Valentin Gledel, Jonathan Narboni, and Théo Pierron. *A note on the flip distance between non-crossing spanning trees.* arXiv preprint arXiv :2303.07710, 2023.